

$$\lim_{x \rightarrow \frac{\pi}{2}} x \tan x - \frac{\pi}{2} \operatorname{Sec} x = \lim_{x \rightarrow \frac{\pi}{2}} \frac{x \operatorname{Sin} x - \frac{\pi}{2}}{\operatorname{Cos} x}$$

$$x - \frac{\pi}{2} = t \Rightarrow x = \frac{\pi}{2} + t \Rightarrow \lim_{t \rightarrow 0} \frac{(\frac{\pi}{2} + t) \operatorname{Cos} t - \frac{\pi}{2}}{-\operatorname{Sin} t}$$

$$\lim_{t \rightarrow \frac{\pi}{2}} \frac{\frac{\pi}{2} \cos t + t \cos t - \frac{\pi}{2}}{-\sin t}$$

$$= \lim_{t \rightarrow \frac{\pi}{2}} \frac{-\frac{\pi}{2} (1 - \cos t) + t \cos t}{-\sin t} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{-\frac{\pi}{2} \times \frac{1}{2} t^2 + t}{-\sin t} = -1$$